Illusion induced overlapped optics

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Abstract: The traditional transformation-based cloak seems like it can only hide objects by bending the incident electromagnetic waves around the hidden region. In this paper, we prove that invisible cloaks can be applied to realize the overlapped optics. No matter how many in-phase point sources are located in the hidden region, all of them can overlap each other (this can be considered as illusion effect), leading to the perfect optical interference effect. In addition, a singular parameter-independent cloak is also designed to obtain quasi-overlapped optics. Even more amazing of overlapped optics is that if N identical separated in-phase point sources covered with the illusion media, the total power outside the transformation region is N2I0 (not NI0) (I0 is the power of just one point source, and N is the number point sources), which seems violating the law of conservation of energy. A theoretical model based on interference effect is proposed to interpret the total power of these two kinds of overlapped optics effects. Our investigation may have wide applications in high power coherent laser beams, and multiple laser diodes, and so on.

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1. Introduction

Transformation optics (TO) theory based on the coordinate transformation and form-invariant of Maxwell's equations, as a useful method to manipulate the propagation path of electromagnetic waves has attracted much attention in the past few years [1–16]. One of the most typical applications of transformation optics is invisible cloak, which is theoretically proposed by J. B. Pendry [1] and U. Leonhardt [2], respectively. Subsequently, many other kinds of cloaks such as square cloaks, elliptic cloaks, diamond-shaped cloaks, arbitrary-shaped cloaks, open cloaks, the general open-closed cloaks and so on [17–25], were also theoretically proposed. Furthermore, microwave, terahertz and optical cloaks have been experimentally demonstrated [26–34]. Recently, complementary media (anisotropic and inhomogeneous negative index materials) has been introduced into the transformation optics, and many interesting TO devices were realized. For example, T. Yang *et al*., have proposed the electromagnetic superscatterer, which means that a small object seems bigger than its geometric size by covering this complementary media [35]. Such a kind of complementary media can also be applied to design external cloaks, in which an object outside the invisible media (complementary media) cannot be detected for the incident electromagnetic waves [36]. Many other novel phenomena of superabsorber, illusion optics, and tunable electromagnetic gateways were also studied based on the complementary media [37–39]. Due to the space fold effect of complementary media, Y. Xu *et al*., theoretically demonstrated the overlapped optics, in which multi-objects appeared like only one (or multi-sources overlapped with each other) [40]. Later, shifting media was proposed in our previous work to realize the overlapped effect [41]. Based on conformal transformation, H. Chen *et al*., designed a new conformal lens with isotropic positive index materials to transform multi-sources into one, resulting in the overlapped effect [42]. In addition, isotropic radiation of a directive source [43] is experimental verification by P. H. Tichit *et al*., and it is very close to the overlapped effects. Meanwhile, as opposite to overlapped optics, W. X. Jiang *et al*., propose the ghost illusion effect and experimental demonstration this effect by transforming one object into several virtual objects [44].

However, all of the above schemes of the overlapped optics have two obvious problems unsolved as yet. For the negative index material-dependent overlapped optics, if we want to make *N* point sources overlap each other, we need to at least *N-1* overlapping transformation devices, and it is very complex for experimental demonstration. For conformal lens-dependent overlapped optics, the transformation media parameters are depending on the number of the point sources. That is to say, different numbers of point sources overlap each other need to design transformation media with different parameters. Another problem is the explanation of the total power of the overlapped effect. For example, if two identical separated point sources overlapped with each other by using the overlapping transformation devices, the total amplitude increases by a factor of 2, but the total power by a factor of 4 (not by a factor of 2). So, if *N* point sources overlapped with each other, the total power increases by a factor of N^2 (not by a factor of *N*). In this paper, motivated by these two unsolved problems, we propose the illusion induced overlapped optics, in which no matter how many in-phase sources in the hidden region, they can overlap each other without needing to *N-1* overlapping transformation devices and changing the transformation media parameters. Furthermore, based on interference theory, we propose a theoretical model to explain the total power of the overlapped effect.

2. Perfect overlapped optics

In this section, we will investigate the perfect overlapped optics based on the traditional cylinder-shaped cloak. The fundamental principle of such a kind of cloak is that a circular shaped region in the virtual space is squeezed into a circular shaped shell in the physical space, resulting in a hidden region. In other words, a point in free space is expanded as a finite region (the hidden region). The corresponding coordinate transformation can be written as follows [3]:

$$r' = \frac{b-a}{b} r + a, \qquad (1)$$

where b and a represent the external and internal radius of the shell, as shown in Fig. 1(a). Obviously, a special singular point can be found in Eq. (1), *i.e.*, when $r' = a$, $r = 0$. Here, $r = 0$ is mapped into $r = a$, which indicates that one point is mapped into countless points located at $r = a$. That is to say, countless points located at $r' = a$ in physical space is equivalent to the case that these countless points perfect overlapped with each other and located at $r = 0$ in the virtual space. Furthermore, if the hidden region ($r \leq a$) is embedded with Epsilon-Near-Zero (ENZ) materials [45], all of the point sources in the region $r \leq a$ overlap each other, leading to the perfect overlapped optics (Here, all of the point sources overlapping with each other can be considered as the illusion effect, and the corresponding transformation media wrapped around these point sources can be thought of as illusion media).
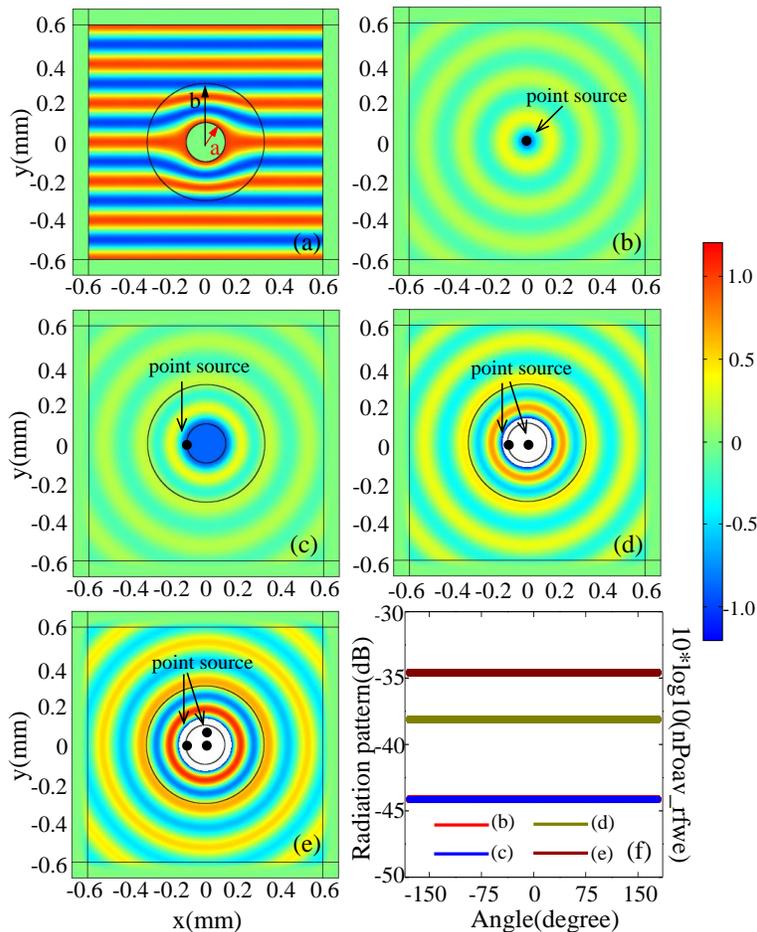

Fig. 1. Electric field distribution of the cloaking and overlapped optics: (a) cloaking. (b)one point source located at (0, 0) without invisible media. (c) one point source located at (−0.1mm, 0) covering with invisible media. (d) two point sources located at (−0.1mm, 0) and (0, 0.1mm), respectively and covering with invisible media. (e) three point sources located at (0, 0), (−0.1mm, 0) and (0, 0.05mm), respectively and covering with invisible media. (f) the corresponding far-field patterns of (b), (c), (d), and (e) at $r = 0.5$mm. The source frequency is 1.5 THz.

The corresponding transformation media parameters from Eq. (1) can be expressed:

$$\bar{\bar{\varepsilon}} = \bar{\bar{\mu}} = \begin{bmatrix} \varepsilon_r \cos^2\theta + \varepsilon_\theta \sin^2\theta & (\varepsilon_r - \varepsilon_\theta)\cos\theta\sin\theta & 0 \\ (\varepsilon_r - \varepsilon_\theta)\cos\theta\sin\theta & \varepsilon_\theta \cos^2\theta + \varepsilon_r \sin^2\theta & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix},$$ (2)

where $\varepsilon_r = \mu_r = \dfrac{r-a}{r}$, $\varepsilon_\theta = \mu_\theta = \dfrac{r}{r-a}$, $\varepsilon_z = \mu_z = \left(\dfrac{b}{b-a}\right)^2 \dfrac{r-a}{r}$ and $\theta = a\tan(y/x)$. $a = 0.1$ mm, and $b = 0.3$mm.

Figure 1 shows the numerical simulation of cloaking and overlapped optics effects based on *Comsol Multiphysics* with PML boundary condition. For the cloaking effect as shown in Fig. 1(a), the plane wave with frequency of 1.5THz moves from up to down. The structure parameters are as follows: $a = 0.1$mm, and $b = 0.3$mm. By embedding the invisible media (plug parameters of a and b into Eq. (2)) in the shell ($a < r < b$), the object in the hidden region ($r \leq a$) is perfectly hidden, as shown in Fig. 1(a). Figure 1(b) depicts the electric field distribution of one point source (with frequency of 1.5 THz and current intensity I = 0.0003A) located at (0, 0) in free space ($\varepsilon = \mu = 1$), while Fig. 1(c) shows the electric field distribution of an identical point source located at (−0.1mm, 0) and covering with the illusion media. Although, both of these two identical in-phase point sources in Figs. 1(b) and 1(c) located at different positions, they appear the same field distributions, which demonstrate that the point source in Fig. 1(c) located at (−0.1mm, 0) is equivalent to the case of a point source located at (0, 0) in the virtual space. Therefore, both of them have the same far-field pattern, as shown in Fig. 1(f) of the red and blue lines, respectively. Figures 1(d) and 1(e) display the electric field distributions of two identical in-phase point sources (located at (0, 0) and (−0.1mm, 0)) and three identical in-phase sources (located at (0, 0), (−0.1mm, 0) and (0, 0.05mm)), respectively (all parameters of the point sources in Figs. 1(d) and 1(e) are as follows: f = 1.5THz, I = 0.0003A). These point sources located at different places overlap each other, and the corresponding field is obviously enhancement compared to the one point source case (Such a perfect overlapped optics effects can be applied to high power coherent laser beams or multiple laser diodes due to the field enhancement). From the far-field patterns in Fig. 1(f) of green and wine lines, it is clear that the total power of these two (or three) identical point sources located at different places and covered with invisible media is four (or nine) times of just one point source. So, if N identical point sources overlap each other, the total power is $N^2 I_0$ (not $N I_0$), and I_0 is the power of just one point source. In addition, no matter how many point sources located at the hidden region ($r \leq a$) they can also overlap each other, and it is not shown in this paper.
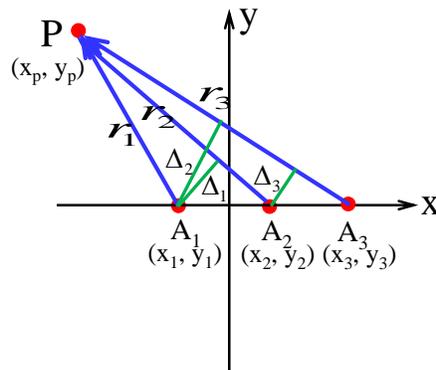

Fig. 2. Schematic of three separated point sources meet at point *p*.

Now, we give the theoretical explanation of the total power of the overlapped point sources. Taking three point sources for example (shown in Fig. 2), three separated point

sources located at A_1, A_2 and A_3 meet at point P. The corresponding electric field vector of each point source can be written:

$$\begin{cases} E_1(r_1,t) = A_1 \exp[i(k_1 \cdot r_1 - \omega_1 t + \delta_1)] \\ E_2(r_2,t) = A_2 \exp[i(k_2 \cdot r_2 - \omega_2 t + \delta_2)], \\ E_3(r_3,t) = A_3 \exp[i(k_3 \cdot r_3 - \omega_3 t + \delta_3)] \end{cases} \qquad (3)$$

where A_1, A_2, A_3 and k_1, k_2, k_3 are the amplitude and wave vector of each point source. ω_1, ω_2, ω_3 and δ_1, δ_2, δ_3 are frequency and initial phase of these three point sources, respectively.

According to the principle of light wave superposition [46], resultant vector of these three point sources at point p is

$$E(r,t) = E_1(r_1,t) + E_2(r_2,t) + E_3(r_3,t) \qquad (4)$$

The light intensity of these three point sources at point p can be expressed as

$$\begin{aligned} I =& <E \cdot E^*> = E_1 \cdot E_1^* + E_2 \cdot E_2^* + E_3 \cdot E_3^* + <\mathrm{Re}\{2E_1 \cdot E_2^*\}> \\ &+ <\mathrm{Re}\{2E_1 \cdot E_3^*\}> + <\mathrm{Re}\{2E_2 \cdot E_3^*\}> \\ =& A_1^2 + A_2^2 + A_3^2 + 2A_1 A_2 <\cos[(k_1 \cdot r_1 - k_2 \cdot r_2) + (\delta_1 - \delta_2) - (\omega_1 - \omega_2)t]>, \\ &+ 2A_1 A_3 <\cos[(k_1 \cdot r_1 - k_3 \cdot r_3) + (\delta_1 - \delta_3) - (\omega_1 - \omega_3)t]> \\ &+ 2A_2 A_3 <\cos[(k_2 \cdot r_2 - k_3 \cdot r_3) + (\delta_2 - \delta_3) - (\omega_2 - \omega_3)t]> \\ =& I_1 + I_2 + I_3 + 2(I_{12} + I_{13} + I_{23}) \end{aligned} \qquad (5)$$

where I_1, I_2, I_3 corresponding to the light intensity of each point source. I_{12}, I_{13}, and I_{23} are the interference light intensity of these three point sources. Therefore, if more than one point source meets in free space, the total light intensity should contain the coherent parts of light intensity, not just the superposition of the light intensity of each point source.

If these three point sources are identical and in-phase with each other ($A_1 = A_2 = A_3 = A$ $k_1 = k_2 = k_3 = k$, $\delta_1 = \delta_2 = \delta_3$ and $\omega_1 = \omega_2 = \omega_3$), the total light intensity can be written

$$I = I_0(3 + 2\cos\varsigma_1 + 2\cos\varsigma_2 + 2\cos\varsigma_3), \qquad (6)$$

where $I_0 = |A|^2$, $\varsigma_1 = k \cdot (r_1 - r_2) = k \cdot \Delta_1$, $\varsigma_2 = k \cdot (r_1 - r_3) = k \cdot \Delta_2$, $\varsigma_3 = k \cdot (r_2 - r_3) = k \cdot \Delta_3$. Δ_1, Δ_2, and Δ_3 are the optical path difference (OPD) between these three point sources at point p, as shown in Fig. 2. Furthermore, the total power of these three point sources at point p is proportional to the light intensity of Eq. (6). If the OPD between these three point sources are zero (the traditional completely coherent condition), the total light intensity is $9I_0$.

For N separated identical in-phase point sources, the light intensity of these separated point sources at point p can be written as

$$\begin{aligned} I &= NI_0 + 2I_0 \sum_{m \neq n} \cos(k \cdot r_m - k \cdot r_n) \\ &= NI_0 + 2I_0 \sum_{m \neq n} \cos(\varsigma_{mn}) \end{aligned} \qquad (7)$$

where $\varsigma_{mn} = k \cdot (r_m - r_n) = k \cdot \Delta_{mn}$ and $m = 1, 2, \cdots N$, $n = 1, 2, \cdots N$. When $\varsigma_{mn} = 0$ ($\Delta_{mn} = 0$), the total light intensity is $I = N^2 I_0$ (not NI_0) due to the perfect coherent condition of $\Delta_{mn} = 0$.
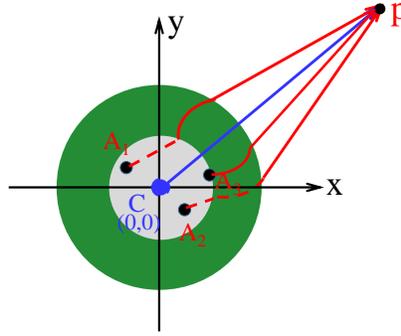
Fig. 3. Schematic of three separated point sources coating with invisible media meet at point *p*.

According to the above discussion, our overlapped optics effects can be explained by using the above interference theory. We also taking three in-phase point sources located in the hidden region for example (Fig. 3). Although these three in-phase point sources are located at different positions, they are virtually overlapped with each other in the virtual space. In physical space, electric fields of these three point sources (A_1, A_2 and A_3) propagate along the three red lines, and it is equivalent to the case that these three point sources propagate along the blue line from the same position C (0, 0) and meet at point *p* in the virtual space, as shown in Fig. 3. Therefore, at point *p*, the OPD between these three separated in-phase point sources at anywhere is $\Delta_1 = \Delta_2 = \Delta_3 = 0$ (due to the perfect overlapping of these three point sources). So, the total light intensity of these three separated point sources is

$$I = I_0(3 + 2\cos\varsigma_1 + 2\cos\varsigma_2 + 2\cos\varsigma_3) = 9I_0 \qquad (\varsigma_1 = \varsigma_2 = \varsigma_3 = 0) \qquad (8)$$

Here, we want to emphasize that the total light intensity of these three in-phase separated point sources covered with illusion media is $9I_0$ (not $3I_0$) without needing to satisfy the traditional completely coherent condition ($\Delta = m\lambda$, m is integer), in physical space. According to Eq. (8), if two in-phase point sources overlapped with each other, the total light intensity (or power) is four times of just one point source. So, according to Eq. (7) when N in-phase point sources overlapped with each other, the total light intensity is N^2I_0 (not NI_0) due to the coherent effect (the total power is proportion to the total light intensity). In traditional case, N separated identical in-phase point sources are corresponding to the light intensity of $I \in [0, N^2I_0]$ ($I = 0 \sim N^2I_0$) (when $\Delta_{mn} = m\pi$, $I = N^2I_0$ while when $\Delta_{mn} = (m+1/2)\pi$, $I = N^2I_0$). Here, when the N separated identical in-phase point sources covered with the illusion media, the total light intensity at any part of the free space is $I = N^2I_0$. Therefore, the total power of the N separated identical point sources covered with the illusion media is larger than that of the traditional case (without covering with illusion media), which seems to violate the law of conservation of energy (not really violate the law of conservation of energy). Here, it completely comes from the perfect coherent effect (That is to say, the N separated identical point sources covered with illusion are completely overlap each other, leading to the total light intensity of $I = N^2I_0$ at any part of the free space).

3. Quasi-overlapped optics

In this section, we study quasi-overlapped optics based on singular parameter-independent cloak shown in Fig. 4. The corresponding coordinate transformation contains two steps: First, the yellow line with length of $2h_1$ is transformed into a longer one with length of $2h_2$. Second, the region II, III, VI, VII and IX is compressed into region II, III, VI, and VII. Here, the coordinate transformation in region I and II can be written as follows (In other region, the coordinate transformation is the same as in region I, or II):
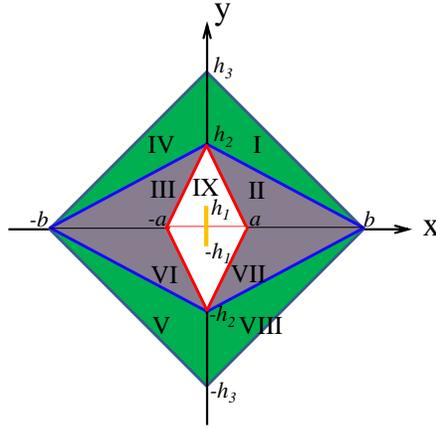
Fig. 4. Schematic of a singular parameter-independent cloak.

In region I:

$$x'' = x, \quad y'' = -\frac{(h_1 - h_2)h_3}{(h_1 - h_3)b}x + \frac{h_2 - h_3}{h_1 - h_3}y + \frac{(h_1 - h_2)h_3}{h_1 - h_3}, \quad z'' = z \tag{9}$$

The material parameter in region is

$$\bar{\varepsilon}_I = \bar{\mu}_I = \begin{bmatrix} \dfrac{h_1 - h_3}{h_2 - h_3} & -\dfrac{(h_1 - h_2)h_3}{(h_2 - h_3)b} & 0 \\[3mm] -\dfrac{(h_1 - h_2)h_3}{(h_2 - h_3)b} & \dfrac{(h_1 - h_2)^2 h_3^2 + (h_2 - h_3)^2 b^2}{(h_1 - h_3)(h_2 - h_3)b^2} & 0 \\[3mm] 0 & 0 & \dfrac{h_1 - h_3}{h_2 - h_3} \end{bmatrix} \tag{10}$$

In region II:

$$x'' = \frac{b - a}{b}x - \frac{a}{h_1}y + a, \quad y = \frac{h_2}{h_1}y, \quad z'' = z \tag{11}$$

The material parameter in region II is

$$\bar{\varepsilon}_{II} = \bar{\mu}_{II} = \begin{bmatrix} \dfrac{(b - a)h_1}{b - h_2} + \dfrac{ba^2}{h_1 h_2 (b - a)} & -\dfrac{(b - a)h_3}{(b - a)h_1} & 0 \\[3mm] -\dfrac{(b - a)h_3}{(b - a)h_1} & \dfrac{bh_2}{(b - a)h_1} & 0 \\[3mm] 0 & 0 & \dfrac{(b - a)h_1}{b - h_2} \end{bmatrix} \tag{12}$$
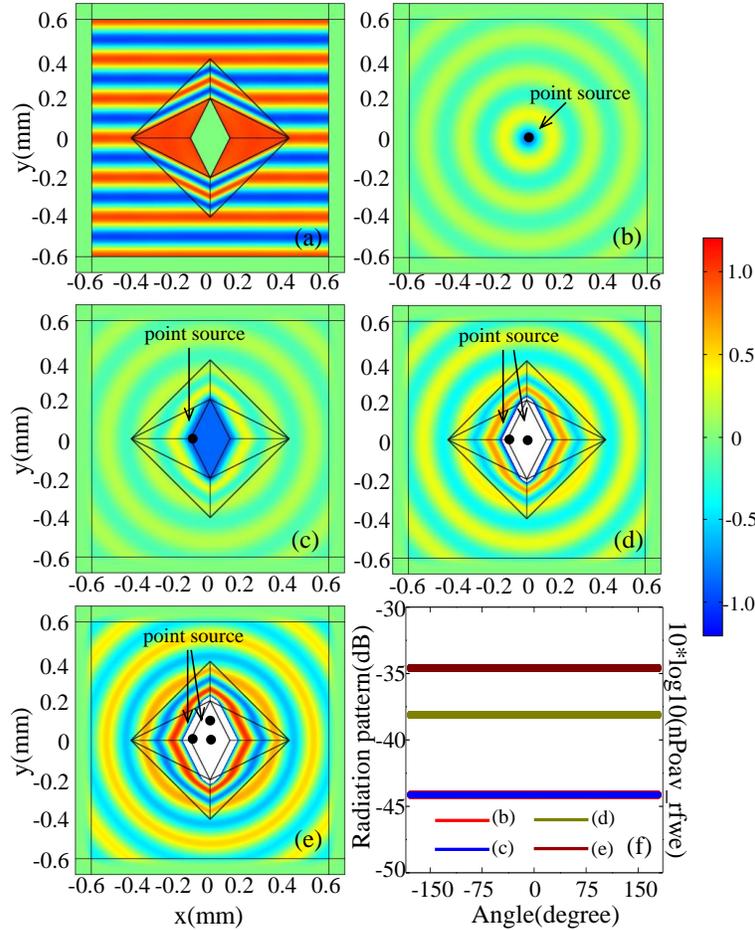
Fig. 5. Electric field distribution of the singular parameter-independent cloaking and overlapped optics: (a) cloaking. (b)one point source located at (0, 0) without invisible media. (c) one point source located at (−0.1mm, 0) covering with invisible media. (d) two point sources located at (−0.1mm, 0) and (0, 0.1mm), respectively and covering with invisible media. (e) three point sources located at (0, 0), (−0.1mm, 0) and (0, 0.1mm), respectively and covering with invisible media. (f) the corresponding far-field patterns of (b), (c), (d), and (e) at $r = 0.5$mm. The source frequency is 1.5 THz.

From Eqs. (10) and (12), it can be found that the parameters in the transformation region are non-singular constants, which means that such a cloak is singular parameter-independent cloak. Such a kind of cloak implies that a short line (yellow line) is extent to a region such as region XI in Fig. 4.

Figure 5 shows the numerical simulations of such a kind of singular parameter-independent cloak and quasi-overlapped optics based on *Comsol Multiphysics* with PML boundary condition. In Fig. 5(a), the incident plane wave with frequency of 1.5THz (the wavelength is 0.2*mm*) propagates from up to down for the structure parameters of $h_1 = 0.015mm$, $h_2 = 0.2mm$, $h_3 = 0.4mm$, $a = 0.1mm$, and $b = 0.4mm$. The object in region XI is hidden when $2h_1 = 0.2mm$ (wavelength of incident electromagnetic wave), and the incident plane wave is bending around the region XI. Figure 5(b) displays the electric field distribution of a point source (with frequency of 1.5 THz and current intensity $I = 0.0003$A) located at (0, 0), while Fig. 5(c) shows the electric field distribution of an identical point source located at (−0.1*mm*, 0) and covering with the illusion media. Both of them appear the nearly same far-field pattern at $r = 0.5mm$, as shown in Fig. 5(f) (red and blue lines). The corresponding

electric distributions of two or three identical separated in-phase point sources located in region XI are shown in Figs. 5(d) and 5(e), respectively(all parameters of the point sources in Figs. 5 (d) and 5(e) are $f = 1.5$ THz, and $I = 0.0003$ A). From these electric field distributions, we can find that these in-phase point sources nearly overlap each other. The total powers (detected at $r = 0.5$ mm) nearly enhance four (Fig. 5(d)) and nine (Fig. 5(e)) times compared to the one point source case, as shown in Fig. 5(f) of yellow and wine lines.

The total power of singular parameter-independent cloak induced overlapped optics can also be explained by using the coherence theory, as demonstrated in Eqs. (6) and (7). In this situation, the fundamental principal of the singular parameter-independent cloak is that a short line is mapped into a finite region. Therefore, any point source in hidden region (region XI-the physical space) is equivalent to the case that a point is located at the short line in the virtual space. If the length of such short line is much smaller than the wavelength of the point source, all of the point sources in this region are nearly overlapped with each other, leading to the quasi-overlapped optics. We can explain the total power of quasi-overlapped optics also by taking three point sources case for example, as shown in Fig. 6. The three separated in-phase point sources (A_1, B_1 and C_1) in region XI coating with the invisible media is mapped into A_2, B_2 and C_2, respectively in the virtual space. The corresponding total light intensity of these three separated point sources at point p is that

$$I = I_0(3 + 2\cos\varsigma_1 + 2\cos\varsigma_2 + 2\cos\varsigma_3) \approx 9I_0 \qquad (\Delta_i \approx 0(i=1,2,3), \quad 2h_1 \ll \lambda). \quad (13)$$

Here, these three separated in-phase point sources (A_1, B_1 and C_1) covered with illusion media can be equivalent to the three separated in-phase point sources (A_2, B_2 and C_2) located at the yellow short line (in virtual space). Because the length of the short line is much smaller than wavelength of these point sources, the OPD at point p of these three separated point sources (A_2, B_2 and C_2) are very small, and it even can be ignored ($\Delta_i \approx 0(i=1,2,3)$). So, the total light intensity of the nearly overlapped three separated in-phase sources is about $9I_0$. If N in-phase point sources are nearly overlap each other, the total light intensity is also about N^2I_0 (see Eq. (7)), and the total power is proportion to the total light intensity.
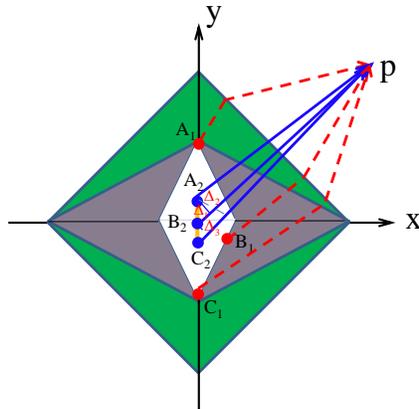

Fig. 6. Schematic of three separated point sources coating with singular parameter-independent invisible media meet at point p.

4. Conclusion

In conclusion, two kinds of cloaks such as normal cylindrical-shaped cloak and singular parameter-independent diamond-shaped cloak are proposed to realize the illusion effects of perfect/quasi-overlapped optics, respectively. No-matter how many in-phase point sources located in the hidden region, all of them completely/nearly overlap each other, and thus enhancing the total field intensity. A theoretical model based on coherence theory is performed to explain the enhancements of the field intensity. Many potential applications,

i.e., high power coherent laser beams, multiple laser diodes and so on, may be achieved by using these two kinds of overlapped optics.

Acknowledgments