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Revealing the THz wave-laser velocity effect during air filamentation via the travelling-wave-antenna model

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During femtosecond laser filamentation in air, the velocity ratio (K) between the terahertz (THz) phase velocity and the laser group velocity plays a crucial role in THz wave generation. However, K is typically assumed to be unity, and its impact has long been overlooked due to the attention paid to the more easily controlled filament length. Here, we investigate the obscured contribution of K to the THz radiation characteristics by using the improved travelling-wave-antenna (TWA) model. It has been found that, under both single- and two-color laser pumping schemes, K significantly determines the far-field spatial distribution of forward or backward THz radiation, as well as the change from Bessel-type to single-ring-type THz emission patterns. These results establish the TWA model as a reliable theoretical tool for studying the mechanisms of THz beam shaping via the designed K . Moreover, for cases of K not being controlled, its value can also be inferred by the proposed TWA model, which could be an effective method to confirm whether the laser ionization front is superluminal or subluminal compared with the generated THz waves. © 2025 Optica Publishing Group. All rights, including for text and data mining (TDM), Artificial Intelligence (AI) training, and similar technologies, are reserved.

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Introduction. Over the past two decades, terahertz (THz) technology has rapidly advanced as a cutting-edge research field, achieving remarkable progress in both fundamental studies and practical applications. Meanwhile, a variety of THz sources have emerged, including quantum cascade lasers [1], photoconductive antennas [2], laser-pumping nonlinear crystals [3], laser ionization of solids [4], and transparent media [5–8]. Beyond these THz sources, the air plasma generated by femtosecond laser filamentation has attracted significant attention in recent years due to its distinctive advantages, such as broad bandwidth and high output intensity [9,10]. In this aspect, the ratio between the THz phase velocity and the laser group velocity (denoted as K) is an essential parameter, no matter in cases of the transition-Cherenkov THz radiation [11–18] and the flying-focus technique [19,20] under single-color fields, or in typical

THz radiation profiles such as Bessel [21] and conical types [22,23] under dual-color fields.

However, due to the tediousness and complexity in directly measuring the laser wavefront propagation velocity [19,20,24,25], it remains challenging to easily determine the value of K in experiments. In this context, previous studies tended to simplify their theories by setting K as 1 [11–15,22,23]. Meanwhile, the plasma filament length (l) closely relates to the THz radiation characteristics and exhibits a similar variation trend with K : under tighter focusing conditions, as l decreases, K decreases accordingly, causing the THz radiation to deviate from the laser axis; conversely, as l increases, K increases, directing the THz radiation along the axis. As a result, current studies have primarily focused on manipulating l [5,26,27], while the role of K has received relatively limited attention. It is also worth noting that due to the uncertainty of the filament's geometric length, imprecision of the l value may obscure the contribution of K in the control of THz beam profiles.

Very recently, with the advancement of laser wavefront control techniques, the significance of K has attracted increasing attention. For instance, under single-color laser field conditions, Fu *et al.* [19,20] demonstrated the controllability of K by using the flying focus method to modulate the laser wavefront propagation velocity, and further verified the K 's critical role in determining the energy distribution of THz radiation in the far field. Furthermore, under dual-color conditions, Zhang *et al.* [21] combined a planoconvex lens and an axicon to generate the plasma filament with a nonuniform axial K distribution, which enabled the observation of far-field Bessel-type THz radiation. Both the above findings suggest that K is no longer a neglectable parameter.

Inspired by these reports, the present work systematically investigates the influence of K on the THz radiation envelopes under the same framework of travelling-wave-antenna (TWA) concept [22,23]. Several representative experimental configurations have been chosen, namely, a single filament in the one- [19,20] or two-color field [21], double counter-propagating filaments [28], and laser-illuminated metal wires [29], respectively. Our results indicate that the TWA model can successfully reproduce the THz radiation patterns related to K as reported in the existing literature, which hence has been demonstrated to

be an effective theoretical tool on the K -control mechanisms of THz radiation.

The improved TWA model. To establish the theoretical framework for this study, we improved the unified TWA equation previously validated by our group [22,23], with K no longer being set as 1 (see also S1 of Supplement 1):

$$|E(\omega, \theta, l, \phi_0)| \propto |\sin \theta| \cdot \left| \frac{e^{i\phi_0} \sin \left[\left(\frac{kl}{2} \right) \left(K - \cos \theta - \frac{\lambda}{2l_d} \right) \right]}{\left(K - \cos \theta - \frac{\lambda}{2l_d} \right)} - e^{-i\phi_0} \frac{\sin \left[\left(\frac{kl}{2} \right) \left(K - \cos \theta + \frac{\lambda}{2l_d} \right) \right]}{\left(K - \cos \theta + \frac{\lambda}{2l_d} \right)} \right|, \quad (1)$$

which could consistently describe the THz radiation characteristics generated by both single- and dual-color laser fields. Here, $E(\omega, \theta, l, \phi_0)$ is defined as the THz electric field, ω is the THz angular frequency, θ is the angle relative to the laser axis, l represents the plasma filament length, and ϕ_0 is the initial phase difference between the fundamental laser and its second harmonic. Additionally, $k = \omega/c$ is the THz wave number in free space, and K indicates the ratio between the THz phase velocity and the laser group velocity. The dephasing length l_d is the distance over which the phase difference between the two colors shifts by π .

To facilitate our subsequent exploration of the THz field generated by the interference of counter-propagating filaments in the single-color laser field, we also derive another improved expression besides Eq. (1). Briefly, as the dephasing length l_d approaches infinity, Eq. (1) simplifies to the single-color case [23]. Then, we adjust θ for the two counter-propagating filaments: for one filament, θ is replaced with $\theta + \pi/2$; and for the other, θ is replaced with $\theta - \pi/2$. The overall THz radiation is formed by $|E'(\omega, \theta, l)| = |E(\omega, (\theta + \pi/2), l) - E(\omega, (\theta - \pi/2), l)|$ as:

$$|E'(\omega, \theta, l)| \propto |\cos \theta| \cdot \left| \frac{\sin \left(\frac{kl}{2} [K + \sin \theta] \right)}{K + \sin \theta} + \frac{\sin \left(\frac{kl}{2} [K - \sin \theta] \right)}{K - \sin \theta} \right|. \quad (2)$$

In following sections, Eqs. (1) and (2) will be used separately to study the K effect on the THz beam profiles.

Controlling the THz spatial distribution by modulating K in the single-color case. Recently, Fu *et al.* [19,20] precisely controlled the laser wavefront velocity in the field of THz radiation from a single-color laser filament, enabling the tunability of K and the flexible shaping of the THz beam envelope. In order to compare with the reported results, we similarly started with a low frequency of 0.1 THz from a 1-mm-long single-color filament under different K values (1:1.8, 1:1, 1:0.6, 1:-1.2, and 1:-0.8), and based on Eq. (1) with $l_d \sim \infty$, we calculated the THz radiation distribution as shown in Fig. 1(a). It can be seen that a positive K directs the THz main lobe along the laser path (0°), while a negative K reverses it. Moreover, as $|K|$ increases, the THz radiation direction gradually approaches the filament axis. These results are consistent with Refs. [19,20]. We also increased the filament length (e.g., to 3 mm), and the TWA-calculated results (not shown) demonstrate the same THz radiation trend with that in Fig. 1(a) of this work and Fig. 3(b) of Ref. [19].

Furthermore, based on the literature [19], we changed the above 0.1 THz to a broadband radiation (0.1–10 THz) with K

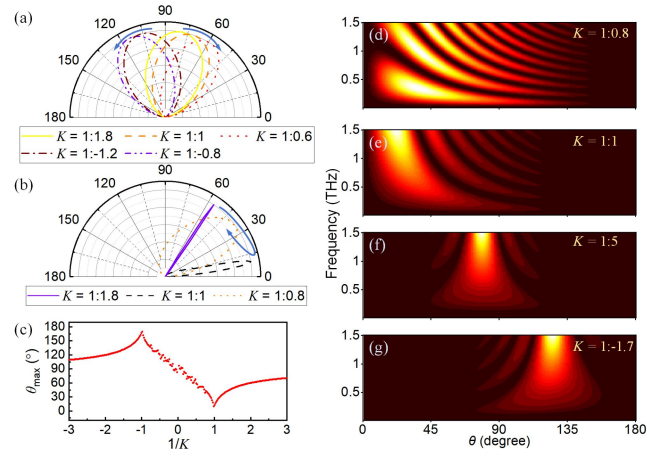


Fig. 1. THz radiation characteristics of a 1-mm-long single-color filament under different K values. (a) At 0.1 THz. (b) For a broadband range (0.1–10 THz). (c) At 5 THz for θ corresponding to the maximum THz radiation. (d)–(g) The THz distribution as functions of frequency and radiation angle.

values of 1:1.8, 1:1, and 1:0.8. It can be observed in Fig. 1(b) that, for $K = 1:0.8$, the THz profile exhibits a broad angular distribution, while for $K = 1:1.8$ or 1:1, the THz radiation is concentrated within a narrow angle, which facilitates a highly directional THz emission and detection. This behavior is also consistent with the results reported in Ref. [19]. However, as $|K|$ increases, the THz direction initially moves closer to the laser axis and then deviates along the arrow in Fig. 1(b). About this point, we take the central frequency of 5 THz in Fig. 1(b) as an example to calculate the relationship between θ of the peak THz output and $1/K (= v/c)$, as shown in Fig. 1(c). It can be seen that at $1/K = 1$ (i.e., $v = c$), θ reaches its minimum. This result not only agrees with Fig. 5(a) of Ref. [20], but also can be easily understood from a physical perspective: when the THz wave propagates at the same speed as the laser ($v = c$), the THz phase matching and constructive interference along the laser propagation axis are most readily achieved, causing the THz radiation to concentrate near the laser axis ($\theta = 0$) and thus resulting in the smallest radiation angle. This could provide a clear physical picture for why the THz radiation angle is the smallest at $K = 1:1$ compared with $K = 1:1.8$ and $K = 1:0.8$ in Fig. 1(b), and further verify Eq. (1).

More calculations at different frequencies (0–1.5 THz) are shown in Figs. 1(d)–1(g), following Ref. [20]. Here, the THz spectral distributions are as a function of the radiation angle θ with different K values (1:0.8, 1:1, 1:5, and 1:-1.7). The first three figures (Figs. 1(d)–1(f)) show that as K decreases from 1:0.8 to 1:5, the THz radiation becomes more concentrated in a narrower angular range, consistent with the trend in Fig. 1(b). And in Fig. 1(g) with $K = 1:-1.7$, the THz emission direction is opposite to the laser propagation, in agreement with Fig. 1(a) for negative K values. All these calculations in Fig. 1 based on the TWA model confirm the significant K effect on the profile control of THz beams. It is also worth noting that, since the TWA model doesn't consider the high-frequency decay of the THz radiation efficiency from a laser filament, the THz distributions towards 1.5 THz in Figs. 1(d)–1(g) are stronger than those in Figs. 5(b)–5(e) of Ref. [20]. To address this issue, we introduced a simple window function to Figs. 1(d)–1(g),

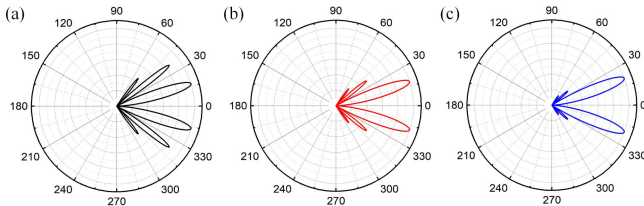


Fig. 2. THz radiation patterns in the two-color case with a K variation along the plasma filament: (a) from 1:1.004 to 1:1.019, (b) from 1:1.006 to 1:1.016, and (c) fixed at 1.

which then could well match the reported results in S2 of Supplement 1.

Controlling the Bessel-type THz spatial distribution by modulating K in the two-color case. To further validate the K 's role, we examined the effect of a non-uniform K along the dual-color plasma filament on the THz radiation, following Ref. [21]. Specifically, based on Eq. (1), THz radiations were calculated at 0.7 THz from a 55-mm-long filament with a 25-mm dephasing length (l_d), where the laser ionization front velocity increased from $1.006c$ to $1.016c$ (c is the light speed in vacuum) along the plasma filament given by a combined focusing of lens and axicon. Thus, K varied between 1:1.006 and 1:1.016. And we also changed this K range into a large one as 1:1.004–1:1.019, or a smaller one as 1:1–1:1 (constant) along the filament.

Corresponding calculation results are shown in Fig. 2, inside which one can see the multiple side lobes characteristic of a Bessel-type THz beam in Fig. 2(a), resulted from the spatial interference from plasma sources with different K values along the laser filament. Then, as the K range was reduced as shown in Fig. 2(b), the interference effect weakened, reducing the side lobes while still maintaining some Bessel beam features. More verifications of this THz-Bessel beam can be found in S3 of Supplement 1. Finally, the interference effect is no longer considered when K becomes constant as 1 along the filament. In this case, as shown in Fig. 2(c), the THz radiation pattern changes from a multiple side lobes structure to conical propagation within a specific angle. The above TWA-predicted results confirm that the axial distribution of K along the filament plays a key role in steering the THz beam, consistent with the literature [21].

Determination of the K value via the TWA model. In previous sections, the TWA model has been validated to accurately predict the THz radiation characteristics with designed K values under both single- and dual-color field conditions. In this section, for cases of K not being controlled, its value has been inferred by fitting the experimental/simulated THz radiation results with the proposed TWA model, which could be an effective method to determine K and the laser ionization front velocity in intense field experiments. However, before computing the K value, other parameters in the TWA formula should be firstly decided, among which the filament length l might be the fuzziest one since the plasma filament is not a geometrical cylinder. On the other hand, intrinsic correlations come between l and K , as mentioned in the introduction. Therefore, imprecision in the l value may obscure the contribution of K , which then cannot be accurately determined. Now, the question becomes: in which case l can be easily decided? and two exemplified cases are as follows.

(1) THz radiation generated by two counter-propagating filaments. The first example involves a counter-propagating fil-

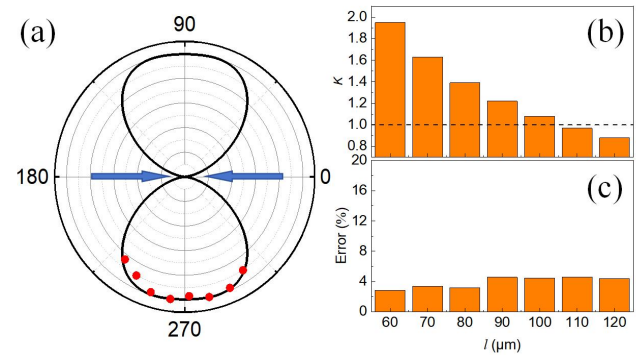


Fig. 3. In case of a counter-propagating filament array, (a) when fitting the THz data in the literature [28] with the TWA model, (b) K is retrieved as a function of l , with (c) the fitting errors (RMSE).

ament array, where two laser pulses travel in opposite directions and interact. In this case, the effective l for THz generation is evaluated by the laser interaction length, which corresponds to the transmission distance of the laser pulses within the duration. For example, in Ref. [28], when two 200-fs laser pulses collide in a counter-propagating manner as shown by the blue arrows in the inset of Fig. 3, l can be estimated as $(200\text{--}400)\text{ fs} \times 3 \times 10^8\text{ m/s}$, resulting in $l = 60\text{--}120\text{ }\mu\text{m}$. Then, we can employ the TWA model (Eq. (2)) to calculate the K value by fitting the 2.1-THz experimental data in Ref. [28], which is primarily emitted perpendicular to the laser axis as shown as red circles in Fig. 3(a).

When the TWA-fitting line (black) closely follows the THz data (red) in Fig. 3(a), the K values are obtained and plotted in Fig. 3(b) with fitting errors (RMSE) as shown in Fig. 3(c) (see also S4 of Supplement 1 in spherical coordinates). It can be seen that K decreases as l increases, but its variation remains mostly above 1 (horizontal dashed line in Fig. 3(b)). This indicates that in Ref. [28], the laser propagation didn't exceed the THz speed. Normally, this superluminal or subluminal issue is fuzzy or difficult to be decided in an easy way. Now, the TWA model offers a new solution. Besides, more significance of this result is provided in Discussion.

(2) THz radiation generated by laser-illuminated metal wires. For the second example (inspired by Ref. [29]), please see S5 of Supplement 1.

Discussion. Note that the transition-Cherenkov radiation model [17] has been widely accepted to explain the far-field conical THz radiation generated by single-color laser filaments in gases. And the theoretical premise of this model requires that the speed of the charged particle exceeds THz wave in the same medium. However, the same research team later pointed out in Ref. [18] about their theory that "superluminality is not strictly necessary". This has raised a question: whether the Cherenkov-like conical THz radiation observed in common-lens-focusing experiments originates from a superluminal or subluminal laser ionization front? To answer this question, remind ourselves that in Fig. 3, we performed THz calculations under the conventional lens focusing condition with a focal length of 2 inches for each filament from the counter-propagating filaments [28]. And the K values were achieved as basically greater than 1, indicating that the speed of laser ionization front is slower than THz waves in air.

This result is meaningful, since it not only is a simple method to judge a superluminal or subluminal laser ionization front benefited from the proposed TWA model, but also could clarify the

aforementioned fuzzy experimental picture that, the laser front is indeed subluminal and it can still effectively produce a far-field conical THz radiation from a single filament (and 90-degree off-axis THz radiation from two counter-propagating filaments). Hence, the TWA model is proved to be versatile to both manage (Figs. 1 and 2) and predict (Fig. 3) K in laser-induced THz generation experiments. On the other hand, the TWA model has also been successfully applied to study the THz radiation phenomena in nongaseous transparent media (e.g., water film) [22], further highlighting the accuracy and applicability of this model in revealing the mechanisms of THz emission in related fields.

Conclusion. Based on the re-worked TWA model, we systematically studied the influence of K on THz radiation characteristics. First, under single-color field, the modulation of THz emission directionality by varying K was revealed. Subsequently, under dual-color field, the effect of nonuniform K distributions along the filament on far-field THz radiation is explored, unveiling the change from a Bessel- to a single-ring-type THz radiation. Furthermore, by analyzing cases involving counter-propagating filaments and laser-illuminated metal wires, the variation trend of K with effective filament length was deduced, confirming the experimental fact that the laser ionization front propagates at subluminal velocities under common 2-inch focal length conditions.

Through these investigations, it is demonstrated that the suggested TWA model could accurately capture the significant K effect on THz radiation characteristics in situation of various experimental configurations, verifying its effectiveness as a theoretical tool for studying the underlying mechanisms associated with K , and laying a foundation for future precise control of THz beam profiles based on K engineering. For example, since tuning K can change the emission direction of THz waves, turning it from forward to off-axis or even backward (Fig. 1(a)), and the strength (or angular range) of the radiation directivity can also be adjusted (Fig. 1(b)), this represents a flexible THz beam-steering approach with 360° freedom. It can be applied to scenarios such as THz base-station scanning broadcast, 3D reflective THz imaging of object contours, and point-to-point directional THz communication, among others.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

Supplemental document. See Supplement 1 for supporting content.

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